AGEC 2025

Challenges in the inversion of large 3-D resistivity and I.P. data sets

M.H.Loke, K. Frankcombe, T. Unrau

Least-squares inversion

Inversion :- Converting field data into a resistivity (I.P) subsurface model.

Most commonly used method for inversion of large 3-D data sets.

$$\left(J^T R_d J + \lambda F_R\right)\Delta q_k = J^T R_d g - (\lambda F_R + \alpha I)(q_k - q_R)$$

q is the model resistivity(I.P) to be determined. ***g*** is the data misfit. ***J*** is the Jacobian matrix that connects ***q*** and ***g***. ***λ***, ***F*** and ***R*** are different settings to stabilise the inversion. To determine the model ***q***, we must (1) calculate ***g*** and ***J*** using the finite-element method, (2) solve the equation.

Problems : If the survey covers a large areas, with many data points and model values, conventional methods exceed the computer memory and time.

Surveys are usually conducted by small companies with limited budget.

How to fit all the calculations into a PC?

Problems and Solutions

1. Problem : Size of Jacobian matrix ***J*** might be hundreds of GB or TB.

   Solution : Reduce size of ***J*** using 1D wavelet compression.

2. Problem : Rectangular finite-element mesh with millions of nodes, exceeds PC capacity.

   Solution : (a) Use non-structured mesh. However, must not conflict with (1).

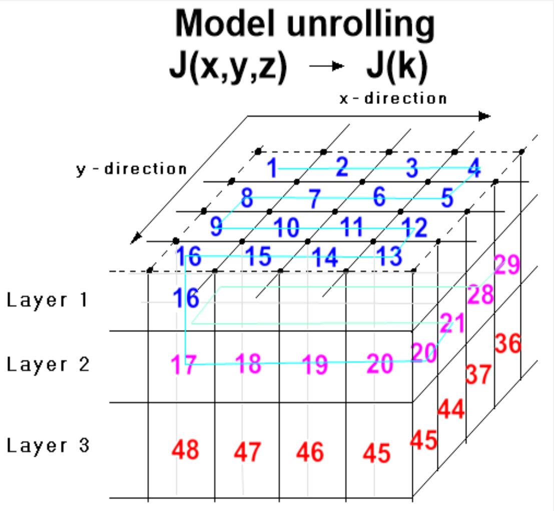
   (b) Subdivide model into number of segments. Can only be used if no remote electrodes.

3. Problem : I.P. surveys, need to calculate apparent I.P. as well.

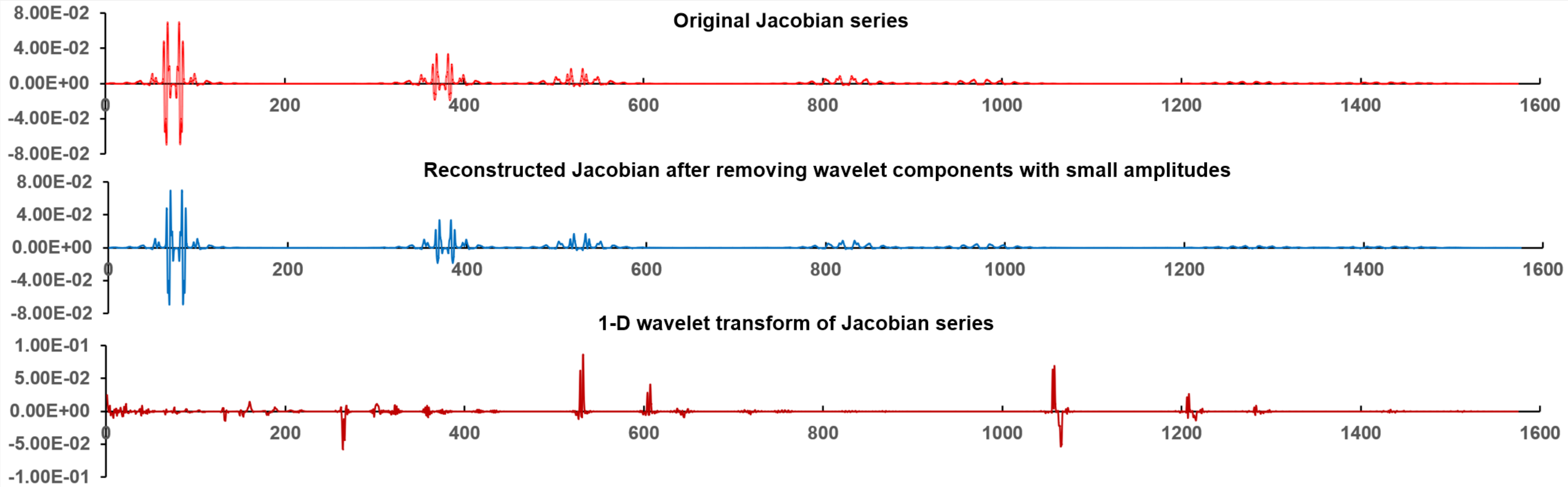
   Solution : Two methods to calculate apparent I.P. Pros and cons.

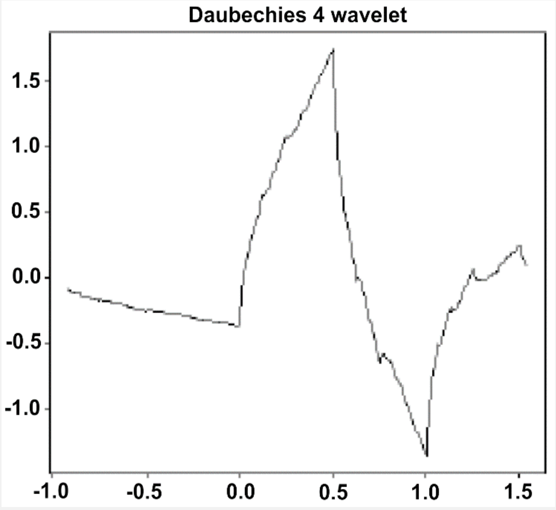
Size of Jacobian matrix *J*

Size is *m*n*, *m*=number of data points, *n*=number of model cells. If m and n are 500,000, RAM required is 931GB well above PCs. Use rectangular arrangement of model cells with 1-D wavelet transform. Wavelet transform has shape similar to Jacobian series.



The 1-D wavelet transform of a Jacobian of a single measurement has most components with amplitude of less than 1% of the highest amplitude. Average difference of reconstructed Jacobian with highest amplitudes from original series is less than 1%. Reduces size of Jacobian matrix by more than 100 times.

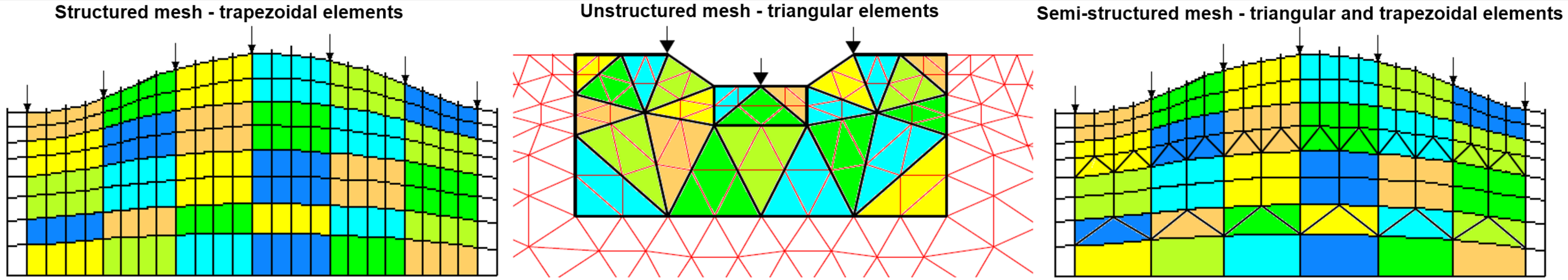




Finite-element mesh

Finite-element method preferred, can easily model topography. Direct solution method can be hundreds of time faster than iterative methods. PC resources needed.

Memory ~ $d\log_2 d$, Calculation time ~ $d^{1.5}$, d=number of nodes in FEM mesh.



Structured mesh – Structured layout of trapezoidal model cells and FEM mesh. Large number of nodes.

Unstructured mesh – Unstructured arrangement of triangular model cells and FEM mesh. Small number of nodes. Wavelet transform compression of Jacobian less effective ~ 2 times.

Semi-structured mesh – Structured arrangement of model cells with semi-structured FEM mesh. Reduce number of nodes by 4 to 10 times. Wavelet transform compression of Jacobian ~ 100 times. Combines advantages of structured model grid and unstructured FEM mesh.

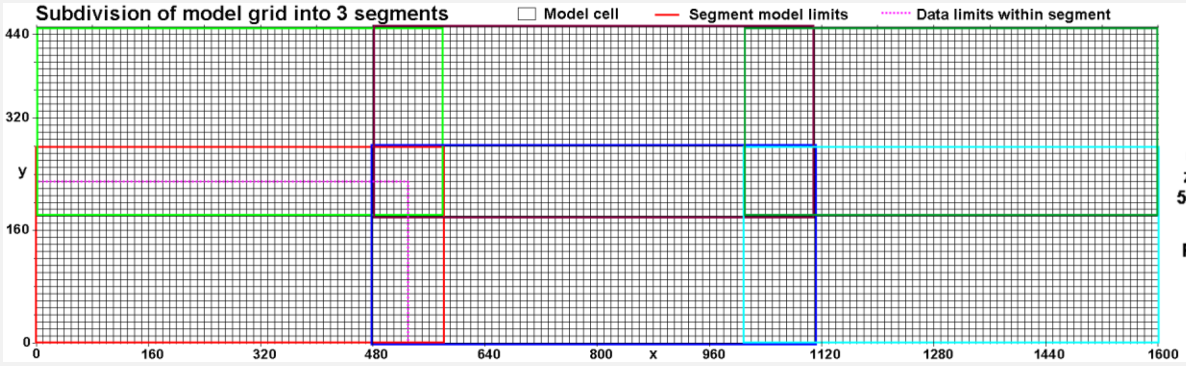
Have your cake and eat it also!

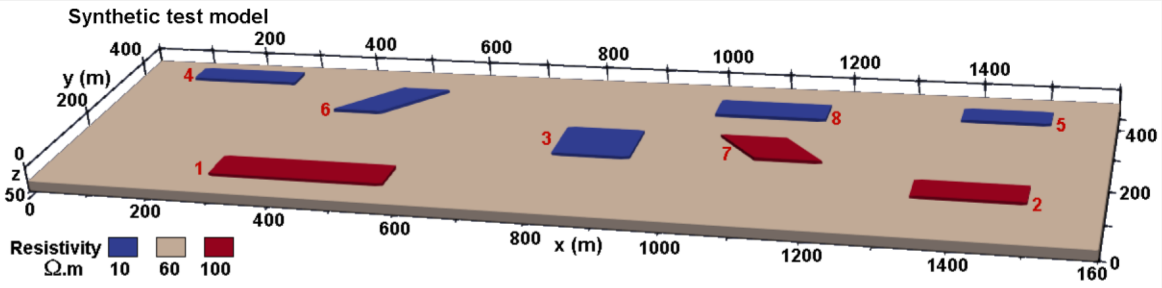
Segmentation

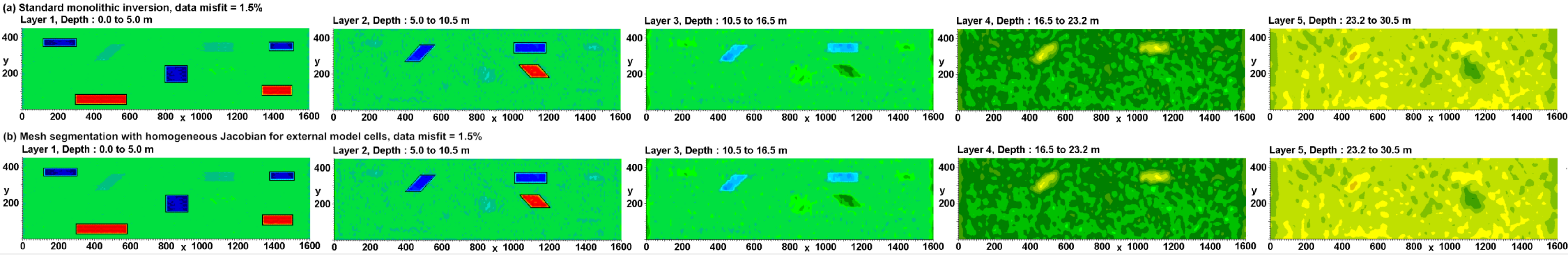
The finite–element mesh can be subdivided into a number of segments if the maximum length of an array is much smaller than the size of the survey area. Anomalies more than several array lengths away have a small effect of the measured potentials. Use overlapping segments to minimise edge effects. The Jacobian values for cells outside the segment can be approximated by the homogeneous half-space Jacobian. A single inversion using the entire model grid is used.

Example model with 8 anomalies. No significant differences between inverse models using apparent resistivity values and Jacobian calculated using a FEM single mesh and with 6 segments.

Limitation : Surveys with remote electrode (eg. offset pole-dipole, vector) that has to be taken into account.







I.P. model calculation methods

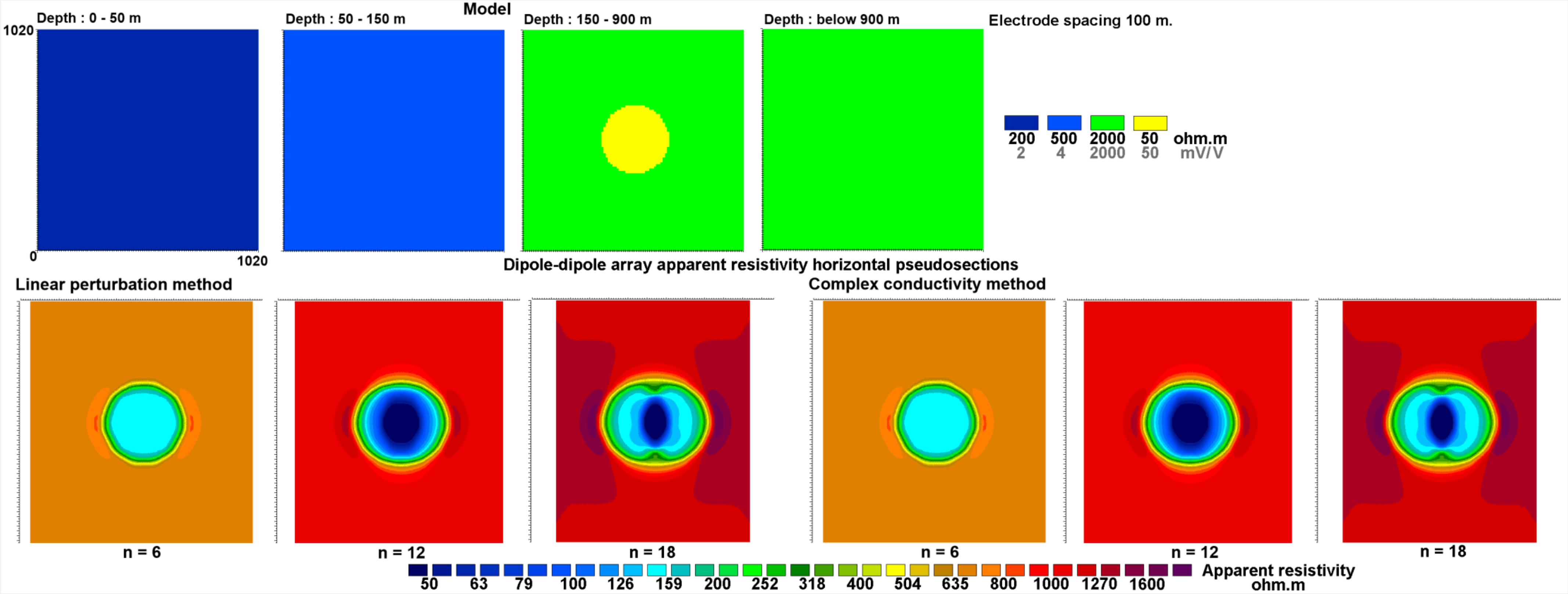
Linear perturbation method :- $m_a = [\phi(\sigma_{IP}) - \phi(\sigma_{DC})]/\phi(\sigma_{DC})$

Faster and requires less memory, but can have problems with round-off errors.

Complex conductivity method :- $\sigma = \sigma_{DC} - i\, m \sigma_{DC}\,, \quad \phi = \phi_r + i\phi_i$

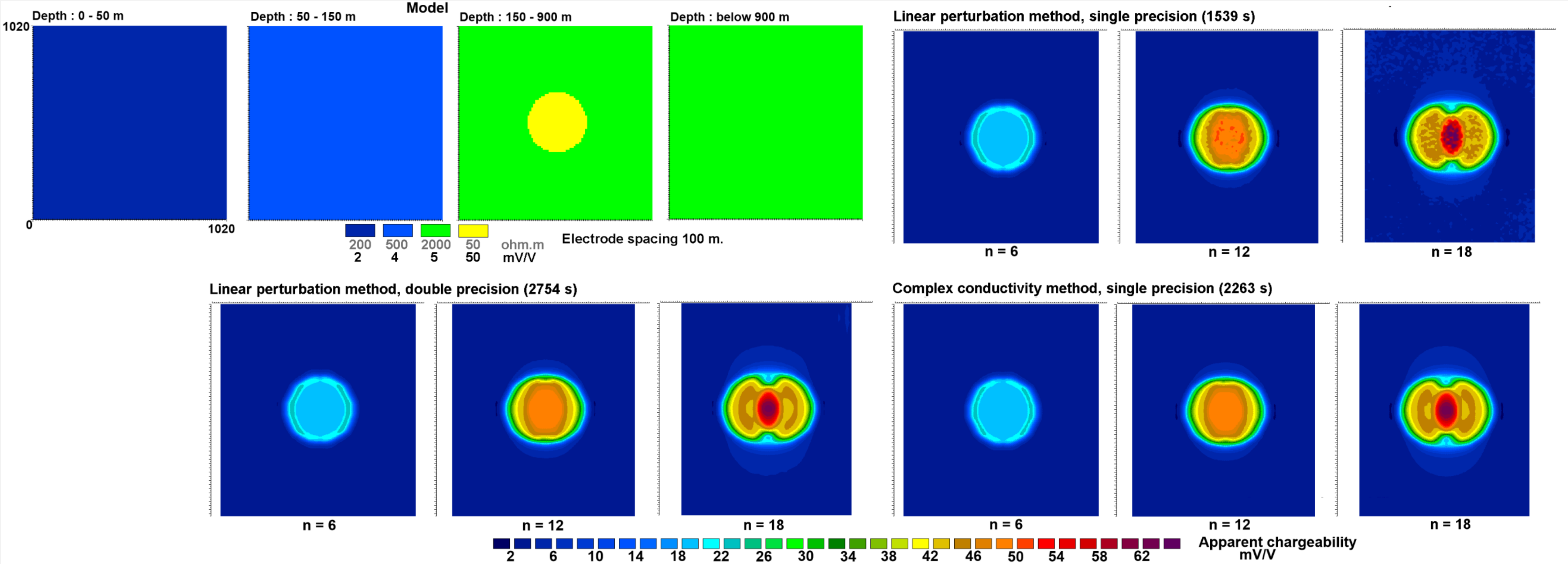
More accurate, but requires twice the PC memory and longer calculation time.

Example 3-D model with large resistivity contrast. Apparent resistivity values almost the same.



I.P. pseudosections

Linear perturbation method with single precision shows some noise for large geometric factors with large model resistivity and I.P. contrasts. Apparent I.P. calculated from difference of two large resistivity potential values. Noise level depends on roundoff errors in solver used, less with double precision calculations. Least noise with the Intel MKL library. Complex conductivity method requires twice as much memory and 50% slower but more accurate.



Kimberlite survey, Canada

Survey using a mobile capacitively coupled system with 10 m dipoles in Northwest Territories, Canada by Aurora Geosciences.

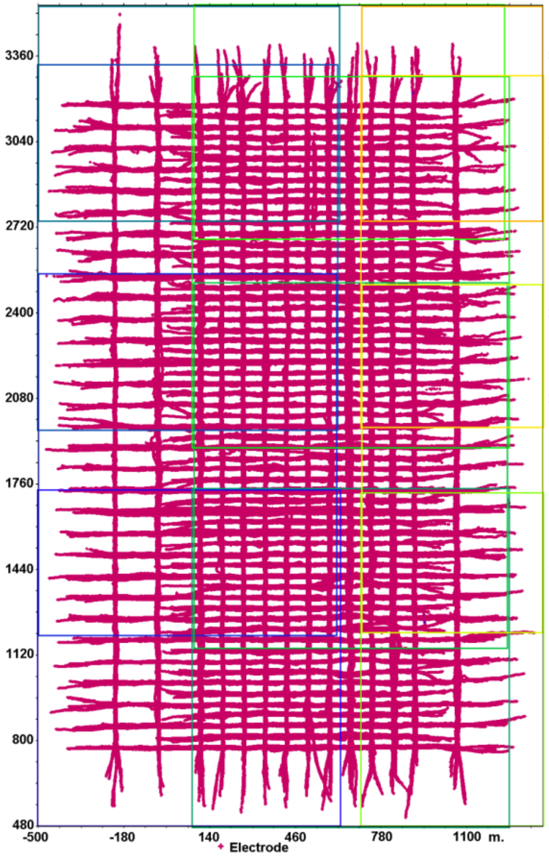
713850 data points, 1250529 electrode positions, 580230 model cells. Full Jacobian : 1500 GB. Compressed Jacobian : 5 GB.

Rectangular mesh : 20 million nodes. Semi-structured mesh, 12 segments, max 1.3 million nodes

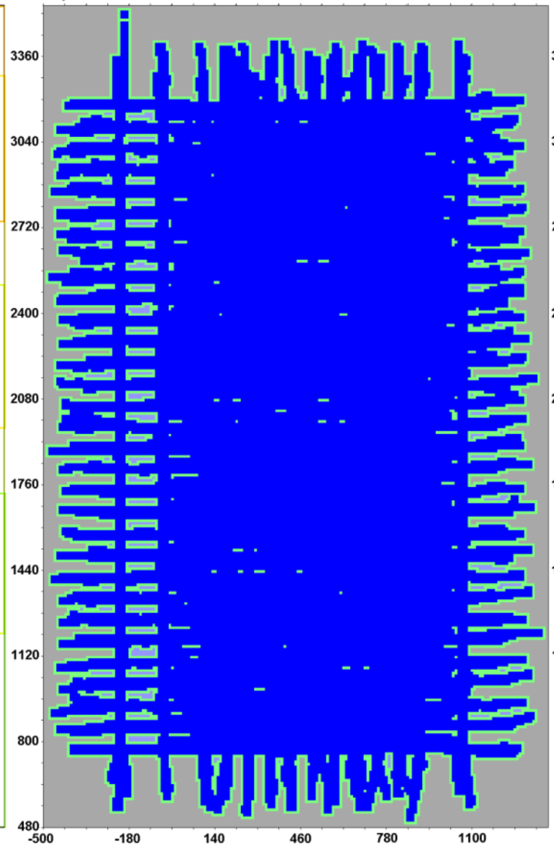
Equipment and weather

Field survey procedure

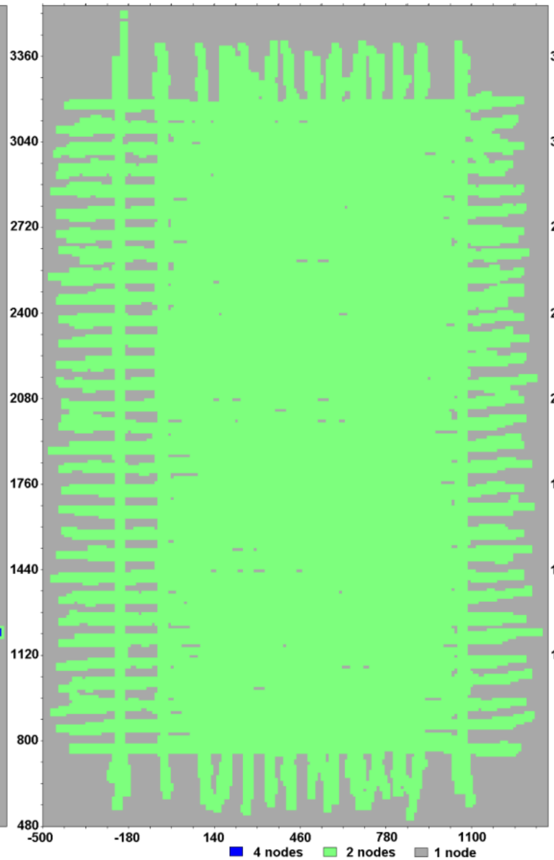
Subdivision into overlapping segments



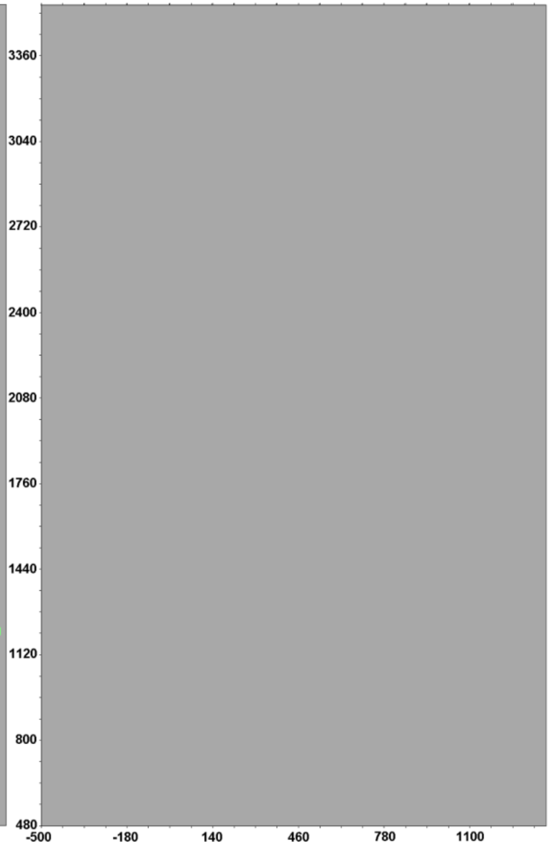
Top level meshes



Mid level meshes



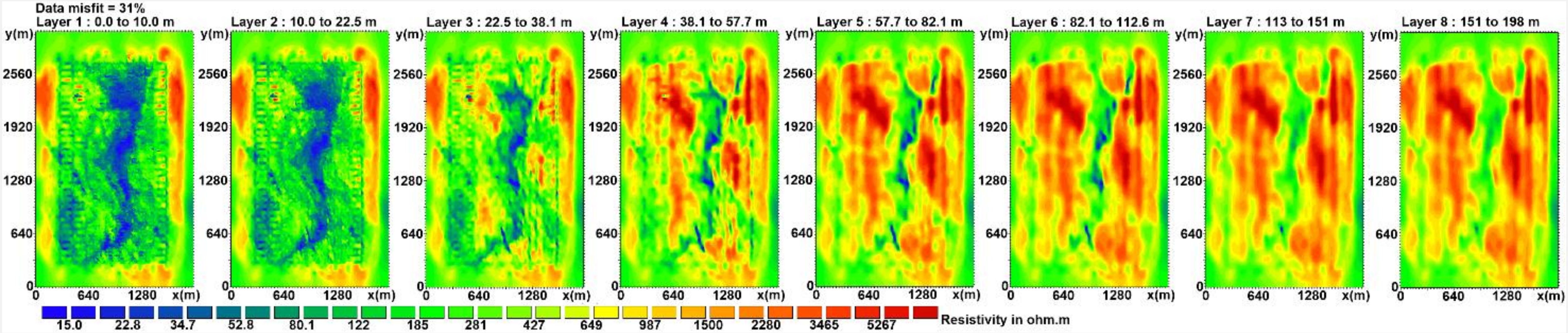
Lower level meshes



Kimberlite survey – model

Part of the survey was over a frozen lake. The inversion time using a PC with a 16-core Intel CPU and 128GB RAM was about 50 hours. The model shows a prominent low resistivity anomaly in the top 6 layers that corresponds to the kimberlite structure. It has a relatively high data misfit of 31% due to the low ground currents generated by capacitively coupled system and errors in the positions of the electrodes measured by a GPS unit on the snowmobile.

Data inversion made possible by combination of subdividing model grid into 12 segments, use of semi-structured finite element mesh and Jacobian compression using 1-D wavelet transform. I.P. option currently being developed for mobile capacitively coupled system.

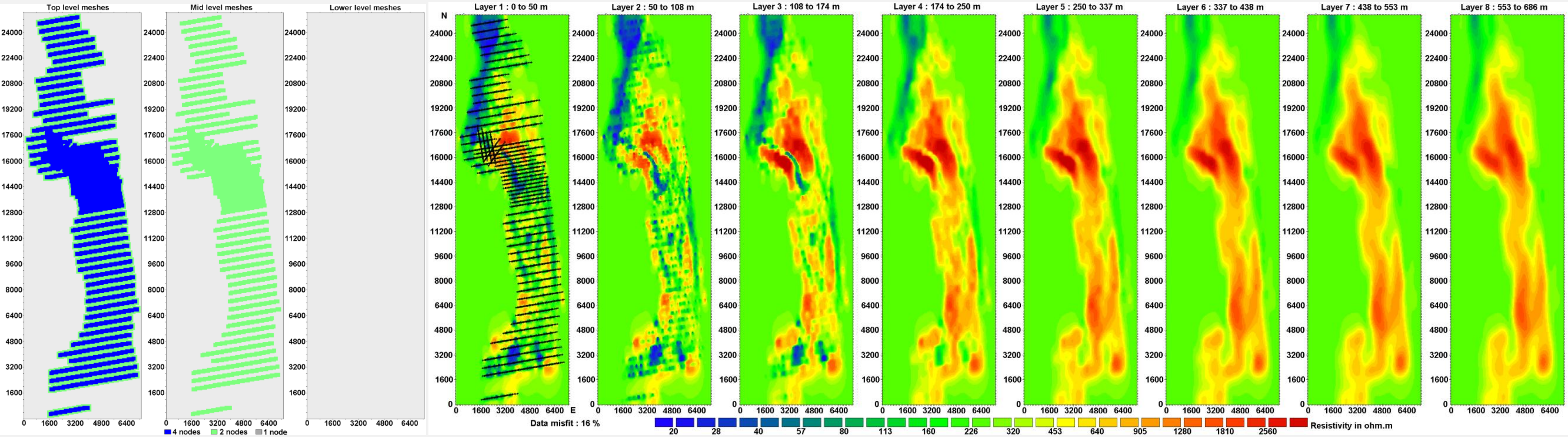


I.P. survey, Queensland – Mesh and resistivity model

A 3-D I.P. survey was carried out over the Roseby copper prospect near Cloncurry, Queensland. The data set has 13735 data points and 3642 electrodes positions. A single finite-element semi-structured mesh was used.

Rectangular mesh : 27 million nodes. Semi-structured mesh : 4 million nodes

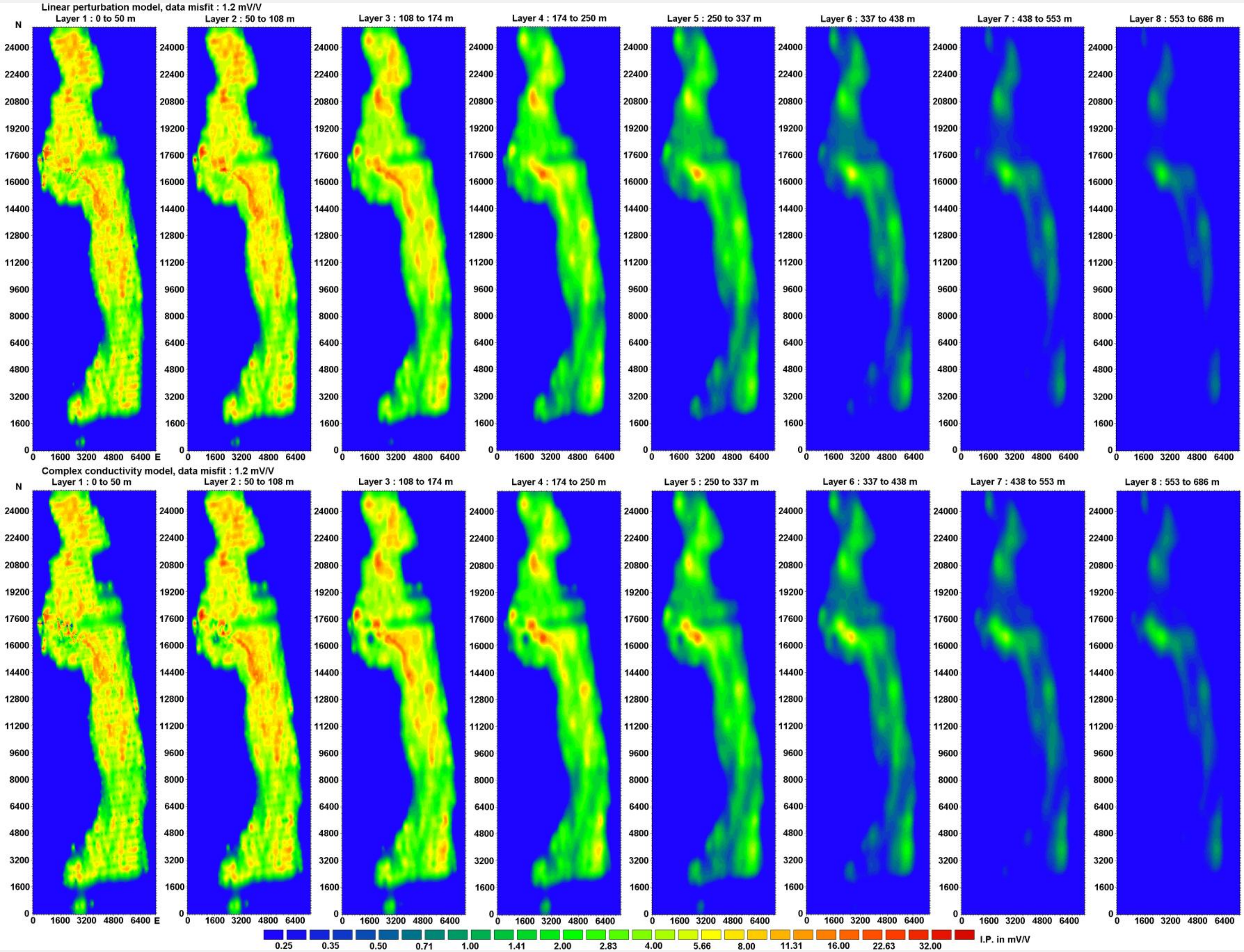
Higher resistivity values between 3000 to 20000 N, with a narrow central low resistivity channel between 13000 to 17000N.



I.P. survey, Queensland – I.P. models

Both linear perturbation and complex conductivity model are very similar. A narrow I.P. anomaly between 12800 and 17600 N at location of low resistivity structure.

Complex conductivity model has boundaries that are slightly sharper, but does not significantly change the main results.



Summary

New geoelectrical survey techniques that cover large areas at greater depths generate enormous data sets that can be a challenge to process with a PC.

The use of different numerical techniques has made it possible to process data sets with millions of data points and model cells with limited computer resources.

Future problems :- SIP? So far, not used routinely in commercial surveys.

Acknowledgements **:-** We would like to thank Darrell Epp of Aurora Geosciences for permission to use Northwest Territories, Canada data set; and Harmony Eva Copper for the Roseby, Queensland, Australia data set.