Fast direct conductivity transforms for TEM systems

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SUMMARY

In 1987, Nekut published in Geophysics a method that used the receding-image approximation of the time domain electromagnetic (TEM) response of a concentric loop system above a half-space to derive a simple, fast, direct transform that calculates resistivity as a function of depth. This method is by far the fastest of published transforms from TEM data to resistivity. Following this example, we make a further simplification that completely eliminates one intermediate step required by Nekut. His intermediate step was used to resolve differences between mirror depth (half the image depth) and the half-space diffusion depth. We simply use the half-space diffusion depth directly in Nekut's receding image method without requiring a mirror-depth calculation and a further calculation of its associated correction. The result is an even faster direct resistivity transform method that exactly matches the published results of Nekut.

A further conceptual advance is immediately clear: the fast direct resistivity transform can be expanded to other common survey geometries such as coincident square- and circular-loop TEM systems. This is achieved through use of the diffusion depth with either direct forward modelling of the half-space or the mirror approximation. We explore this conceptual advantage and give an example of direct resistivity transforms for the Slingram geometry commonly used in electromagnetic surveys.

Key words: apparent conductivity, TEM, central-loop, Slingram, AEM.

NEKUT'S METHOD

When current *I* flows around a circular loop of radius *R*, the magnitude of the magnetic field *H* produced at distance *D* along the central axis of the loop is expressed as

$$H\left(D\right)=\frac{\mu_0 I}{2}\frac{R^2}{\left(R^2+D^2\right)^{3/2}}, \quad (1)$$

where μ_0 is the permittivity of free space. We imagine this loop placed on the surface of the earth as in a traditional electromagnetic sounding. When current *I* is abruptly terminated at time $t = 0$, a secondary magnetic field is induced in the earth beneath the loop. A magnetometer placed in the centre of the loop measures the decay of the vertical component of this secondary magnetic field.

The approximation technique used by Nekut (1987) for the direct inversion of (current step-down) time-domain data relies on a comparison of the vertical magnetic fields produced at the centre of the circular loop. The magnetic field amplitude ratio, $H(D)/H_0$, is found using equation (1), where H_0 is equal to $\mu_0 I/2$:

$$\frac{H\left(D\right)}{H_0}=\frac{R^2}{\left(R^2+D^2\right)^{3/2}}. \quad (2)$$

Nekut uses the approximation that the secondary field induced in the earth can be replaced by a receding image source identical to the ground loop of radius *R*. The image descends through a homogeneous half-space as *t* advances. In his formulation, Nekut replaces *D* by the variable 2δ, in analogy with the image method of solution for a circular loop distance δ above an infinitely conductive thin sheet. When above a homogeneous ground that has conductivity σ, the secondary field can be defined to penetrate to depth $\hat{\delta}$, a time domain analogue of the frequency domain 'skin depth' and which is described by the diffusion depth formula in the time domain, ie

$$\hat{\delta}=\sqrt{\frac{2t}{\mu_0\sigma}}. \quad (3)$$

Equations (2) and (3) are combined with D = 2δ to produce an equation that provides an approximate description of the time-decay of the secondary magnetic field *H*(t). Nekut uses this simple calculation is a comparison to the exact long-wavelength approximation solution (Ward and Hohmann, 1988) provided in the literature. The vertical magnetic field amplitude ratios compared to normalised half-space diffusion depth for both the approximate and exact solutions are shown in Figure 1. Both curves exhibit a $t^{-3/2}$ fall-off in amplitude ratio, typical of a homogeneous half-space.

In order to correct the differences between the exact central-loop solution of the literature and the approximate receding-image solution provided by the vertical magnetic field component at the centre of a circular loop, Nekut applied a correction factor that corrects the mirror depth δ to the diffusion depth $\hat{\delta}$. In essence, Nekut used measured amplitudes to predict a mirror depth δ, then used a correction factor to predict a diffusion depth. Presumably, he did this because the correction factor curve is close to unity and slowly varying and, hence, easily interpolated. It is far simpler and more intuitive to us to simply assume that the amplitude measured is for a half-space and therefore find from the analytic solution a diffusion depth $\hat{\delta}/R$ that matches each data point.

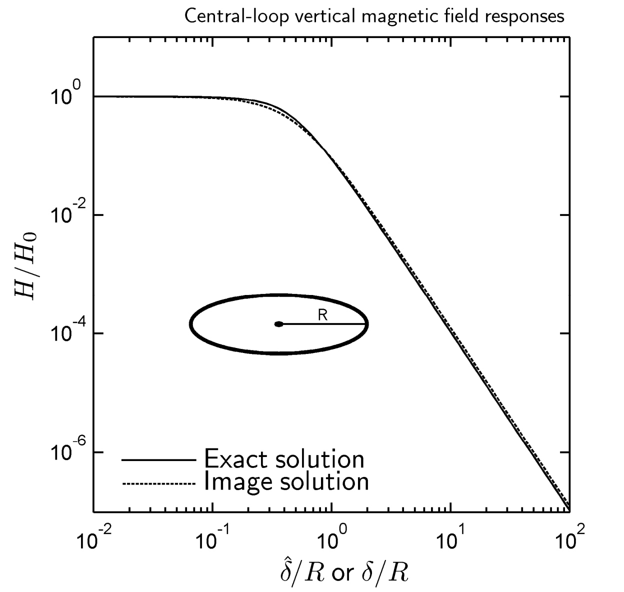

Figure 1. Vertical magnetic field amplitude ratio compared to normalised half-space diffusion depth ratio for both the approximate and exact solutions to a central-loop TEM system (receiver is in centre of loop).

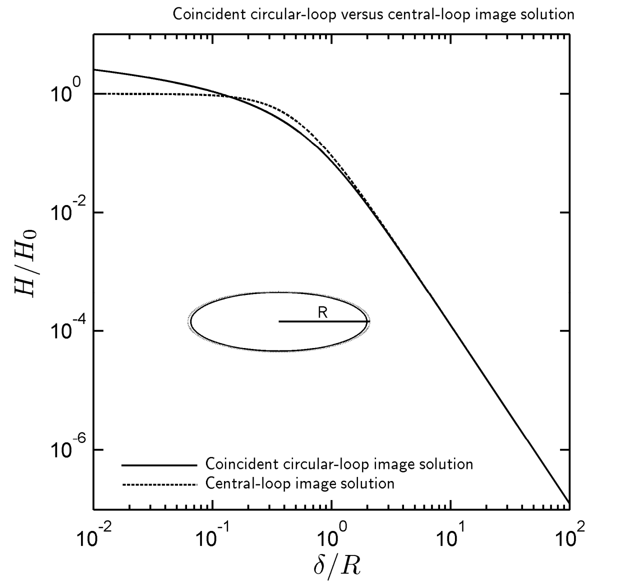

Figure 2. Magnetic field amplitude ratios versus normalised diffusion depth for both the coincident circular-loop and central-loop image solutions.

NEW METHOD FOR DIFFERENT GEOMETRIES

Our direct method avoids Nekut's correction-factor step because we assume that the amplitude ratio measured in a central-loop configuration is the exact amplitude for a half-space. However, the correction factor concept is useful, as it allows us to extend Nekut's method into new geometries for which the exact solution is not analytical: for example, coincident-circular loops or coincident-rectangular loops. Let us consider the coincident circular-loop geometry in which the receiver coil is coincident with the transmitter coil. In this new geometry, we use the measured amplitude ratio to predict a mirror depth δ and assume that this depth is equal to the mirror depth obtained from a central-loop system of an equivalent transmitter area. We then predict the equivalent central loop amplitude. We have then a difference in amplitude between the two geometries. We apply Nekut's discarded correction factor to the predicted central loop amplitude and use the modified amplitude to obtain a diffusion depth that is obtained from a look-up table that connects exact central-loop transient solutions to diffusion depths. Figure 2 shows the variation of amplitude ratios to normalised diffusion depth for both the central-loop exact solution and the coincident circular-loop image solution. This technique can also be extended for coincident rectangular loops.

SLINGRAM GEOMETRY

The extension of Nekut's method to Slingram geometry is the next to be considered. Slingram geometry consists of transmitter and receiver coil pairs (generally) rigidly separated by a distance ρ, Figure 3. In our case, the receiving probes are capable of measuring magnetic fields in the vertical and radial directions.

For our present consideration, the Slingram system is placed on the surface of the earth (z = 0). As in the central-loop case, it is possible to obtain an exact solution for the magnetic field at the receiver position (ρ, 0) due to an abrupt current step-down in a dipole positioned above a homogeneous half-space. The magnetic fields measured at the receivers at time $t = 0$ are compared to the instantaneous magnetic fields at time $t = 0$, and these amplitude ratios are compared to the calculated responses of the exact solution for a homogeneous half-space at different diffusion depths. From the measured and predicted total fields, we predict a diffusion depth and therefore, an apparent conductivity. Figure 4 shows the amplitude ratios for the vertical and radial direction of the secondary magnetic fields versus normalised diffusion depth for the Slingram geometry.

AIRBORNE EXAMPLE

Our goal is to apply our novel application of Nekut's method to the modified Slingram geometry found in the TEMPEST airborne electromagnetic system. In this geometry, the receiver coils are separated from the transmitter by both a radial and vertical offset of (typically) 100 and 30 m, respectively. We apply Nekut's method to obtain apparent

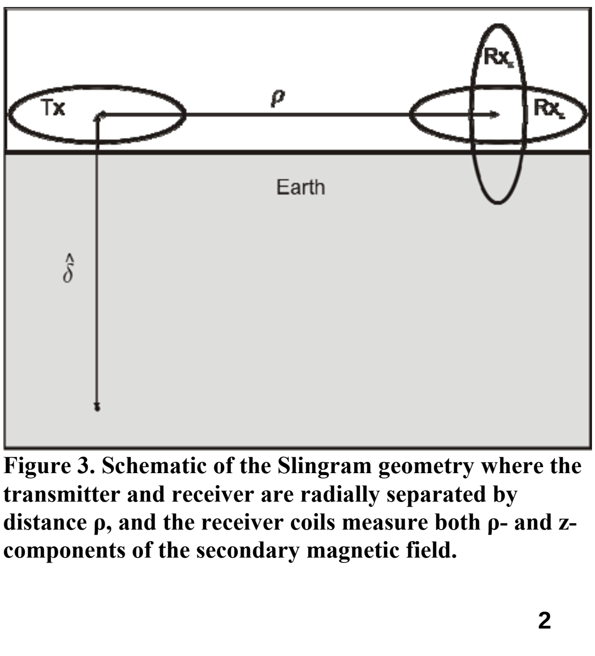

Figure 3. Schematic of the Slingram geometry where the transmitter and receiver are radially separated by distance ρ, and the receiver coils measure both ρ- and z-components of the secondary magnetic field.

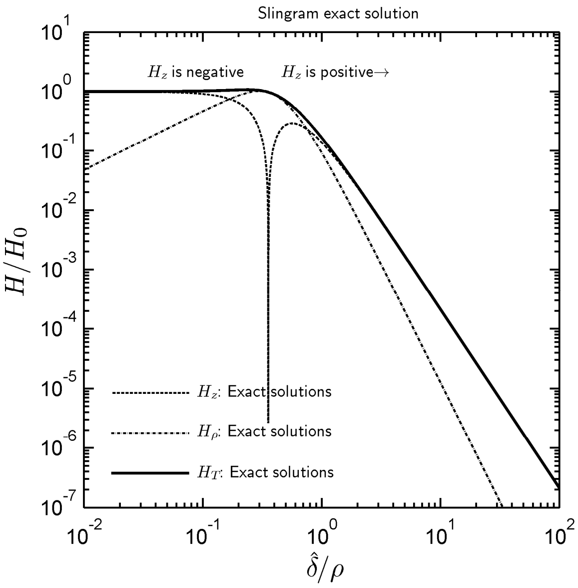

Figure 4. Amplitude ratios versus depth ratios for the Slingram geometry.

conductivity predictions for a small bathymetry testline flown near Mandurah, Western Australia. The flight path and the x-component conductivity-depth image, obtained from processing with EMFlow (Macnae and Lamontagne, 1987), are shown in Figure 5. The bottom panel clearly shows the bathymetry of the seawater, as well as the small sand dunes and intrusion of the saltwater inland slightly to the east of the second dune.

CONCLUSION

Nekut's original method for determining apparent conductivity based on calculating the depth of a receding image based on amplitudes; and then correcting to a diffusion depth using a correction curve, has been replaced with a more efficient one-step process whereby we determine a diffusion depth directly from the exact solution for an abrupt current shut-off of a current source in a central-loop system. We eliminate one step from Nekut's method and use a look-up table of values to determine apparent conductivity directly. This speeds-up the central-loop direct transform significantly.

By modifying Nekut's original method, we extend the direct transform technique to different geometries, namely coincident circular- and rectangular-loop. In this new method, we equate a receding image depth of the new geometry to the receding image depth of the central-loop geometry and then apply Nekut's original correction factor to the amplitude ratios to thereby determine initially a diffusion depth and then an apparent conductivity.

In addition, we also extend our newly modified method to use the exact time-domain solution for a Slingram geometry and apply it to the airborne TEMPEST system. As an example, we produce an apparent conductivity profile for some bathymetry data flown over a testline near Mandurah, Western Australia.

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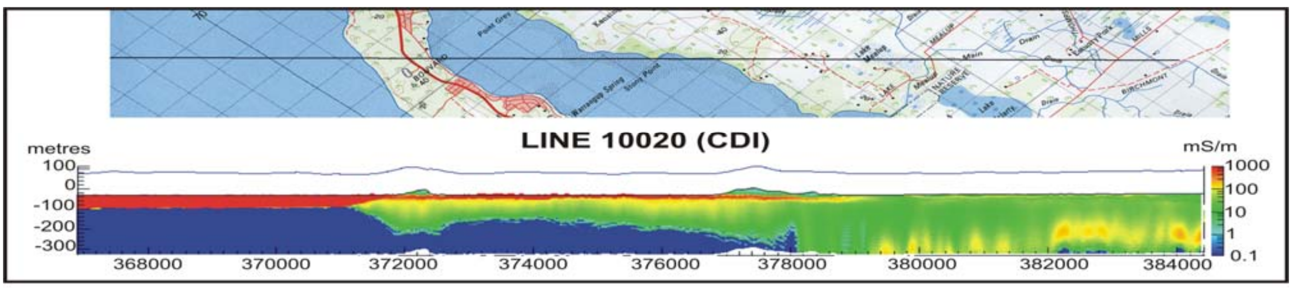
Figure 5. Flight line and conductivity-depth image from bathymetry data acquired by the TEMPEST system near



Mandurah, Western Australia.